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Test MaxEnt in social strategy transitions with experimental two-person constant sum 2 × 2 games

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ABSTRACT

Using laboratory experimental data, we test the uncertainty of social state transitions in various competing environments of fixed paired two-person constant sum 2 × 2 games. It firstly shows that, the distributions of social strategy transitions are not erratic but obey the principle of the maximum entropy (MaxEnt). This finding indicates that human subject social systems and natural systems could share wider common backgrounds.

1. Introduction

The principle of the maximum entropy (MaxEnt) is introduced by Jaynes [19], rooting in Boltzmann, Gibbs and Shannon [20,26]. As a methodology, MaxEnt has gained its wide applications in natural science and engineering. The advantage of this methodology is to provide rich information based on a very limited information. In economics, MaxEnt approach has also gained its wide applications, e.g., in market equilibrium [2,28], in wealth and income distribution [9,31], in firm growth rates [1] and in behavior modeling [30]. Theoretical interpreting or modeling of the distributions of social outcomes with MaxEnt is growing.

Considering the importance of MaxEnt, to carry out laboratory experiments to investigate this fundamental rule is necessary [15]. Only quite recently, entropy is firstly measured in experimental economics systems to evaluate social outcomes by Bednar et al. [4] and Cason et al. [7]. Then, Xu et al. [36] find the human system in laboratory fixed-paired two-person constant-sum 2 × 2 games obey the MaxEnt. To the best of our knowledge, these are almost the total experimental works related to entropy or MaxEnt in social research field till now. In the first experimental investigation in MaxEnt, Xu et al. [36] focus on the *static* observable — distribution and the entropy. A direct one-step-forward question is, in the experimental social interaction systems, whether the *dynamic* observable fits MaxEnt or not?

Answering this question is the main aim of this report. The paper is organized as follows: section two describes the relative notions; section three introduces the experiments and reports of the experimental social transitions; section four provides the MaxEnt prediction relating to the social transitions of the investigated experiments; section five reports the results; discussion and summary are at last.

2. Relative notions

2.1. Two person constant sum 2 × 2 game

Two-person zero-sum games describe situations in which two individuals are absolutely opposite to each other, where one's gain is always the other's loss [21]. Constant sum game is strategically equivalent to zero sum games in mathematical view.

In a two-person constant-sum 2 × 2 game, each player has two strategies. For a row player, the strategy set is (U,D) and for a column player, the strategy set is (L,R). The sum of the two players' payoffs is the same for any outcome. Let S denote the sum of the payoffs of the two players. Any constant sum 2 × 2 game can be written in the form of Fig. 1. A, B, C and D are the payoffs for row player under four combinations of two players' strategies respectively and S-A, S-B, S-C and S-D are for column player respectively. If $(A > C) \cap (B < D) \cap (A > B) \cap (C < D)$ or $(A < C) \cap (B > D) \cap (A < B) \cap (C > D)$ as in [25], there exists an unique mixed strategy Nash equilibrium (MSNE).

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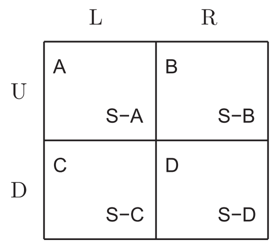

Fig. 1. Payoff matrix of a constant sum game. A, B, C and D are the payoffs for row player under four combinations of two player's strategies respectively and S-A, S-B, S-C and S-D are for column player respectively. S denotes the sum of the payoffs to the two players.

2.2. Social state and observation

The social state x_{ij} [23] can be taken as the combination of two players' strategies, herein i indicates the column player's strategy and j indicates the row player's strategy. Let p be the probability of strategy R for column player and q be the probability of strategy D for row player, the social state can be described by $x_{ij} = (p, q)$. During a game, each player chooses a pure strategy from his own strategy set in a round t, the combination of these two strategies can be taken as a *social state* x_t in that round, $x_t = (p_t, q_t)$. Obviously, there are altogether four possible social states in one round, i.e., (0,0), (0,1), (1,0) and (1,1) indicating LU, LD, RU and RD respectively, and we simplify them as x_{00}, x_{01}, x_{10}, and x_{11}. In Fig. 2(b) and (f), the gray dots represent the social states.

If the game is repeated, *observation* denoted as Ω_{ij} at each state x_{ij} can be accumulated and the results of these games are shown in the last four columns of Table 1.

2.3. Social transition

In this paper, we investigate the social transitions within the strategy states in the strategy space. In a repeated game, for a given round t, the social state is $x_t = (p_t, q_t)$; similarly, the social state in the previous round can be denoted as $x_{t-1}{:=}(p_{t-1}, q_{t-1})$ and in the next round can be denoted as $x_{t+1}{:=}(p_{t+1}, q_{t+1})$. For each given round t, there exists the next round and previous round except the first round and last round in a experimental session. So, there exists a social *forward transition* vector (denoted as T_+) indicating the transition from x_t to x_{t+1}, and a social *backward transition* vector (denoted as T_-) indicating the transition from x_{t-1} to x_t.

In a two-person 2 × 2 game, there are four social states, so there are all 32 transitions (shown in the first column in Table 2), including the four backward (forward) transitions for each of the four states. These 32 transitions should be the samples for MaxEnt testing.

2.4. Distribution of transitions of a given state

During a game, for a given social state, there exists four forward transitions and four backward transitions, respectively. This means that there should exist a distribution of transitions.

For example, Fig. 2(a) is demonstrating the distribution of the transitions of the given state x_{01}, in which the four backward transitions $T_{01\leftarrow00}$, $T_{01\leftarrow01}$, $T_{01\leftarrow10}$ and $T_{01\leftarrow11}$ come from the four states x_{00}, x_{01}, x_{10} and x_{11}, respectively; the blue arrows indicate the directions of transitions and the numerics indicate the related actual frequencies. The distribution of backward transitions $T_{01\leftarrow00}$, $T_{01\leftarrow01}$, $T_{01\leftarrow10}$, $T_{01\leftarrow11}$ are 55, 106, 78, and 193, respectively. Similarly, Fig. 2(e) illustrates the distribution of the forward transitions.

2.5. Aggregated transition of a state

The existence of distribution of transitions of a given state implicates that there are many backward starting points and forward terminal points. So, for a given state $x_{i_0j_0}$, we can get a so-called the mean starting point $\bar{x}_{t-1} = (\bar{p}_{t-1}, \bar{q}_{t-1})$ and a *aggregated backward transition* $\overline{T}_-$ (it is natural that $\overline{T}_- = x_{i_0j_0} - \bar{x}_{t-1}$), and also the mean terminal point $\bar{x}_{t+1} = (\bar{p}_{t+1}, \bar{q}_{t+1})$ and a *aggregated forward transition* $\overline{T}_+$ (it is natural that $\overline{T}_+ = \bar{x}_{t+1} - x_{i_0j_0}$). The aggregated forward transition T_+ is the same as the experimental dynamics observable in literatures (called as change in a given state in Ref. [3] and the mean jump-out vector of a given state in Ref. [33]).

For example, supposing the given state is (0, 1), Fig. 2(b) illustrates the aggregated backward transition $\overline{T}_-$, as the average of

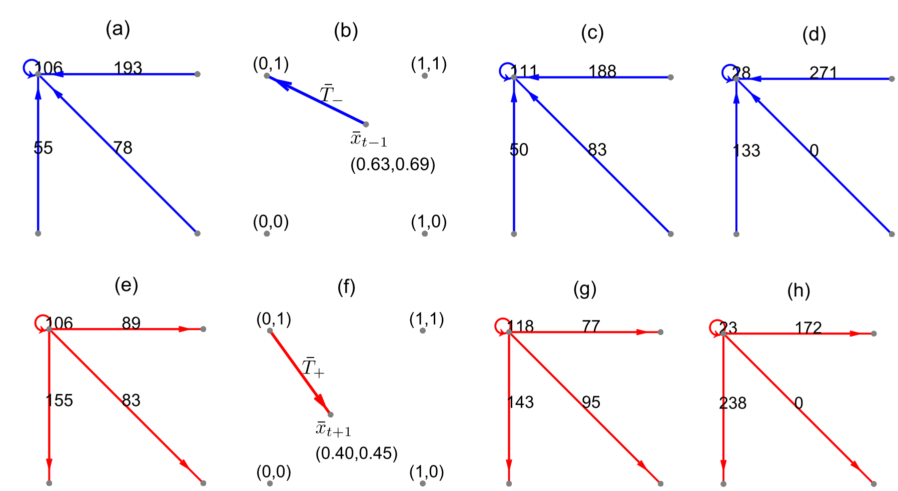

Fig. 2. Transition distribution and aggregated transition. (a) Four actual backward transitions $T_{01\leftarrow00}$, $T_{01\leftarrow01}$, $T_{01\leftarrow10}$, $T_{01\leftarrow11}$ for the given state x_{01} from x_{00}, x_{01}, x_{10}, x_{11}; the blue arrows indicate the directions of transitions; the numerics indicate the frequencies of transitions, respectively; (b) the aggregated backward transition $\overline{T}_-$ and related mean starting point $\bar{x}_{t-1} = (\bar{p}_{t-1}, \bar{q}_{t-1}) = (0.63, 0.69)$; (c) four expected backward transitions for x_{01} from four starting points $x_{0\,0}$, x_{01}, x_{10}, x_{11} with MaxEnt, the blue arrows indicate the directions of transitions and the numerics indicate the related frequencies; (d) an example which is not in agreement with MaxEnt but satisfies the aggregated backward transition constraint in (b);(e) four actual forward transitions $T_{01\rightarrow00}$, $T_{01\rightarrow01}$, $T_{01\rightarrow10}$, $T_{01\rightarrow11}$ from x_{01}, the red arrows indicate the directions of transitions, and the numerics indicate the related actual frequencies of transitions; (f) the aggregated forward transition $\overline{T}_+$ and relative mean terminal point $\bar{x}_{t+1} = (\bar{p}_{t+1}, \bar{q}_{t+1}) = (0.40, 0.45)$; (g) four MaxEnt expected forward transitions; (h) an example which is not in agreement with MaxEnt but satisfies the actual aggregated forward transition constraint. Data in (a) and (e) comes from the experiment — game 1.

Table 1
Parameters and observations of the 11 game.[a]

Game	A	B	C	D	S	Group	Rounds	Ω_{00}	Ω_{01}	Ω_{10}	Ω_{11}
g1	77	35	8	48	100	9	500	994	433	1659	1405
g2	73	74	87	20	100	9	500	1373	250	2401	467
g3	63	8	1	17	100	9	500	664	333	1955	1539
g4	55	75	73	60	100	9	500	643	1611	588	1649
g5	5	64	93	40	100	9	500	548	891	1153	1899
g6	46	54	61	23	100	9	500	1135	706	1729	921
g7	89	53	82	92	100	9	500	502	1840	825	1324
g8	88	38	40	55	100	9	500	353	663	1443	2032
g9	40	76	91	23	100	9	500	1157	860	1366	1108
g10	69	5	13	33	100	9	500	443	465	995	2588
g11	5	0	0	5	5	12	300	837	913	907	931

[a] g1 to g11 indicate game 1 to game 11, respectively. The symbols A, B, C, D and S refer to Fig. 1. Group is the number of the pairs of human subjects playing the games. Rounds is the game repeated times in each pair.

Table 2
Actual frequencies of the transitions of 11 games.

T_{-}	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
$T_{00\leftarrow00}$	464	764	184	314	124	529	143	111	606	116	196
$T_{00\leftarrow01}$	155	52	73	67	68	99	169	95	67	139	241
$T_{00\leftarrow10}$	274	504	327	193	207	382	89	70	362	123	182
$T_{00\leftarrow11}$	102	53	79	66	149	124	98	75	120	63	218
$T_{01\leftarrow00}$	55	86	35	239	213	226	104	11	263	43	149
$T_{01\leftarrow01}$	106	48	89	1054	217	311	1191	264	365	121	216
$T_{01\leftarrow10}$	78	69	100	70	191	86	66	62	85	55	231
$T_{01\leftarrow11}$	193	45	111	245	268	85	482	327	145	247	319
$T_{10\leftarrow00}$	401	446	383	45	51	235	145	169	144	232	281
$T_{10\leftarrow01}$	83	75	99	65	143	86	99	82	99	91	263
$T_{10\leftarrow10}$	1021	1722	1046	258	483	1029	478	858	691	370	191
$T_{10\leftarrow11}$	152	160	424	223	476	380	103	333	434	302	173
$T_{11\leftarrow00}$	74	77	62	45	160	145	110	62	144	52	211
$T_{11\leftarrow01}$	89	75	72	425	463	210	381	222	329	114	193
$T_{11\leftarrow10}$	286	106	482	67	272	232	192	453	228	447	303
$T_{11\leftarrow11}$	958	209	925	1115	1006	332	641	1297	409	1976	221
T_{+}	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
$T_{00\rightarrow00}$	464	764	184	314	124	529	143	111	606	116	196
$T_{00\rightarrow01}$	55	86	35	239	213	226	104	11	263	43	149
$T_{00\rightarrow10}$	401	446	383	45	51	235	145	169	144	232	281
$T_{00\rightarrow11}$	74	77	62	45	160	145	110	62	144	52	211
$T_{01\rightarrow00}$	155	52	73	67	68	99	169	95	67	139	241
$T_{01\rightarrow01}$	106	48	89	1054	217	311	1191	264	365	121	216
$T_{01\rightarrow10}$	83	75	99	65	143	86	99	82	99	91	263
$T_{01\rightarrow11}$	89	75	72	425	463	210	381	222	329	114	193
$T_{10\rightarrow00}$	274	504	327	193	207	382	89	70	362	123	182
$T_{10\rightarrow01}$	78	69	100	70	191	86	66	62	85	55	231
$T_{10\rightarrow10}$	1021	1722	1046	258	483	1029	478	858	691	370	191
$T_{10\rightarrow11}$	286	106	482	67	272	232	192	453	228	447	303
$T_{11\rightarrow00}$	102	53	79	66	149	124	98	75	120	63	218
$T_{11\rightarrow01}$	193	45	111	245	268	85	482	327	145	247	319
$T_{11\rightarrow10}$	152	160	424	223	476	380	103	333	434	302	173
$T_{11\rightarrow11}$	958	209	925	1115	1006	332	641	1297	409	1976	221

[a] g1 indicates game 1, the rest analogize.

the four vectors in Fig. 2(a), is $(-0.63, 0.31)$; Meanwhile the mean starting point $\bar{x}_{t-1} = (0.63, 0.69)$; In Fig. 2(f), $\overline{T}_{+}$, as the average of the four vectors in Fig. 2(e), is $(0.40, -0.55)$ and then $\bar{x}_{t+1} = (0.40, 0.45)$.

3. Data set and experimental transitions

3.1. Experiments and data set

Experimental economics methods are well suited to evaluate theories [15]. In this paper, we use the same data set as Ref. [36] to test the MaxEnt in social strategy transitions. The two-person constant sum 2×2 game includes 11 different parameters (Table 1). From game 1 to game 10, each game consists of nine pairs of subjects, each pair play for 500 rounds while for game 11, the game consists of 12 pairs of subjects, each pair play for 300 rounds. These yield 4500 observed social states in each of game 1 to game 10 and 3600 observed social states in game 11 (for more detail, see Ref. [14,36]).

3.2. Experimental distributions of transitions

According to the data set for each of the 11 games, using the definition in Sections 2.3 and 2.4, we can calculate the actual experimental distributions of backward transitions and forward transitions. The results are summarized in Table 2, which should serve as the targets for testing MaxEnt.

3.3. Experimental aggregated transition for each state

Numerically, $\bar{x}_{t\pm1}$ can be presented by two components $(\bar{p}_{t\pm1}, \bar{q}_{t\pm1})$. Using the definition in Section 2.5, results of all of the components from experiments are shown in Table 3. The vectors, $\bar{x}_{t\pm1}$, are shown in the sub-figures in Fig. 3.

For calculating the theoretical backward (forward) distributions of the transitions from MaxEnt, $\bar{x}_{t-1}$ $(\bar{x}_{t+1})$ should constraint the testing of MaxEnt.

4. Theoretical distributions of transitions from MaxEnt

In this paper, in order to have a deeper insight in the dynamic social observable, we use the aggregated social transitions $(T_{\pm})$ as the constraints for MaxEnt testing. It is clear that for a given state, without MaxEnt, the distribution of backward (forward) transitions can be *arbitrary* even given the constraints $T_{\pm}$ (two examples are given in Section 6).

For a given state $x_{i_0j_0}$, the $\bar{x}_{t-1}$ is assumed to be $(\bar{p}_{t-1}, \bar{q}_{t-1})$ and there is no other information. According to MaxEnt suggested by Jaynes [20], the probability of the backward transitions from the states to $x_{i_0j_0}$ can be expressed, respectively, as

$$p(T_{i_0j_0\leftarrow00}|\bar{x}_{t-1}) = (1-\bar{p}_{t-1})(1-\bar{q}_{t-1})$$
$$p(T_{i_0j_0\leftarrow01}|\bar{x}_{t-1}) = \bar{q}_{t-1}(1-\bar{p}_{t-1})$$
$$p(T_{i_0j_0\leftarrow10}|\bar{x}_{t-1}) = \bar{p}_{t-1}(1-\bar{q}_{t-1})$$
$$p(T_{i_0j_0\leftarrow11}|\bar{x}_{t-1}) = \bar{p}_{t-1}\bar{q}_{t-1}$$

More compactly, the probability of backward transitions from x_{ij} to $x_{i_0j_0}$, can be expressed as,

$$p(T_{i_0j_0\leftarrow ij}|\bar{x}_{t-1}) = \bar{p}^{i}_{t-1}(1-\bar{p}_{t-1})^{1-i}\bar{q}^{j}_{t-1}(1-\bar{q}_{t-1})^{1-j}, \quad (1)$$

in which $\{i,j\} \in \{0,1\}$. Similarly, for $\bar{x}_{i_0j_0}$ and its $\bar{x}_{t+1} = (\bar{p}_{t+1}, \bar{q}_{t+1})$, the probability of the forward transition from $x_{i_0j_0}$ to state x_{ij} can be expressed as

$$p(T_{i_0j_0\rightarrow ij}|\bar{x}_{t+1}) = \bar{p}^{i}_{t+1}(1-\bar{p}_{t+1})^{1-i}\bar{q}^{j}_{t+1}(1-\bar{q}_{t+1})^{1-j}, \quad (2)$$

in which $\{i,j\} \in \{0,1\}$ too.

Comparing to experimental distributions directly, the theoretical probabilities are multiplied by the observation $\Omega_{i_0j_0}$ (referring to Table 1) to gain the theoretical distributions. Fig. 2(c) provides an example to illustrate the theoretical distribution of backward transitions using the $\bar{x}_{t-1}$ in Fig. 2(b) and Eq. (1). Similarly, Fig. 2(g) illustrates the theoretical distribution of forward transitions using $\bar{x}_{t+1}$ in Fig. 2(f) and Eq. (2); Multiplied factor Ω refers to the figure in last four columns of Table 1.

In summary, according to Eqs. (1) and (2), together with Table 3 as constraints, theoretical probabilities of the transitions can be obtained. Multiplied by the observation Ω at the given state, the distribution of the transitions can be obtained and listed in Table 4.

5. Results

To test MaxEnt is to evaluate the goodness of fit between the experimental data (in Table 2) and theoretical data (in Table 4).

Fig. 3 plots the results of theoretical transition frequencies (in horizon, x-axis) and observed experimental transition frequencies (vertical, y-axis). The first figure shows the results for all 11 games, and from second to last is game 1 to game 11, respectively. For each game, there are 32 samples of social strategy transitions. The cycles in blue indicate the backward transitions and the crosses in red indicate the forward transitions. Significantly, all of the backward transition samples (blue cycles) and forward transitions samples (red crosses) are close to the diagonal lines which mean theoretical values from MaxEnt are close to experimental values.

The liner regression results are shown in Table 5. Obliviously, each of the liner regression coefficients is very close to 1 and the $p < 0.001$. Table 5 also provides the 99% C.I. (confidence interval) both for linear regression coefficients and intercept constant. All the lower bound of 99% C.I. of regression coefficients are smaller than but very close to 1 and the upper bound are larger than but also very close to 1; then the equal hypothesis of the two variables cannot be rejected. Meanwhile, for intercept constant (y-intercept, the point where a line crosses the y-axis), none of the p values is smaller than 0.42, all of the lower bound of the 99% C.I. are smaller than 0 and all of the upper bound are larger than 0. Then, the hypothesis that the regression line cross 0 cannot be rejected. These statistical results indicate that the hypothesis that theoretical values statistically equal to experimental observation of the transitions is supported.

In summary, in the laboratory experimental two-person constant sum 2×2 games, the outcome of the social transitions fits MaxEnt. In other words, given the mean vector of the transitions of a given state, the distributions of the all transitions of the given

Table 3
Mean starting point $\bar{x}_{t-1}$ and terminal point $\bar{x}_{t+1}$.

State	$\bar{x}_{t-1}$	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
x_{00}	$\bar{p}_{t-1}$	0.38	0.41	0.61	0.40	0.65	0.45	0.37	0.41	0.42	0.42	0.48
	$\bar{q}_{t-1}$	0.26	0.08	0.23	0.21	0.40	0.20	0.54	0.48	0.16	0.46	0.55
x_{01}	$\bar{p}_{t-1}$	0.63	0.46	0.63	0.20	0.52	0.24	0.30	0.59	0.27	0.65	0.60
	$\bar{q}_{t-1}$	0.69	0.38	0.6	0.81	0.55	0.56	0.91	0.89	0.59	0.79	0.58
x_{10}	$\bar{p}_{t-1}$	0.71	0.78	0.75	0.81	0.83	0.81	0.70	0.83	0.82	0.68	0.40
	$\bar{q}_{t-1}$	0.14	0.10	0.27	0.49	0.54	0.27	0.24	0.29	0.39	0.39	0.48
x_{11}	$\bar{p}_{t-1}$	0.88	0.67	0.91	0.72	0.67	0.61	0.63	0.86	0.57	0.94	0.56
	$\bar{q}_{t-1}$	0.74	0.61	0.65	0.93	0.77	0.59	0.77	0.75	0.66	0.81	0.45
State	$\bar{x}_{t+1}$	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
x_{00}	$\bar{p}_{t+1}$	0.48	0.38	0.67	0.14	0.39	0.33	0.51	0.65	0.25	0.64	0.59
	$\bar{q}_{t+1}$	0.13	0.12	0.15	0.44	0.68	0.33	0.43	0.21	0.35	0.21	0.43
x_{01}	$\bar{p}_{t+1}$	0.40	0.60	0.51	0.30	0.68	0.42	0.26	0.46	0.50	0.44	0.50
	$\bar{q}_{t+1}$	0.45	0.49	0.48	0.92	0.76	0.74	0.85	0.73	0.81	0.51	0.45
x_{10}	$\bar{p}_{t+1}$	0.79	0.76	0.78	0.55	0.65	0.73	0.81	0.91	0.67	0.82	0.54
	$\bar{q}_{t+1}$	0.22	0.07	0.30	0.23	0.40	0.18	0.31	0.36	0.23	0.5	0.59
x_{11}	$\bar{p}_{t+1}$	0.79	0.79	0.88	0.81	0.78	0.77	0.56	0.80	0.76	0.88	0.42
	$\bar{q}_{t+1}$	0.82	0.54	0.67	0.82	0.67	0.45	0.85	0.80	0.50	0.86	0.58

[a] g1 indicates game 1, the rest analogize.

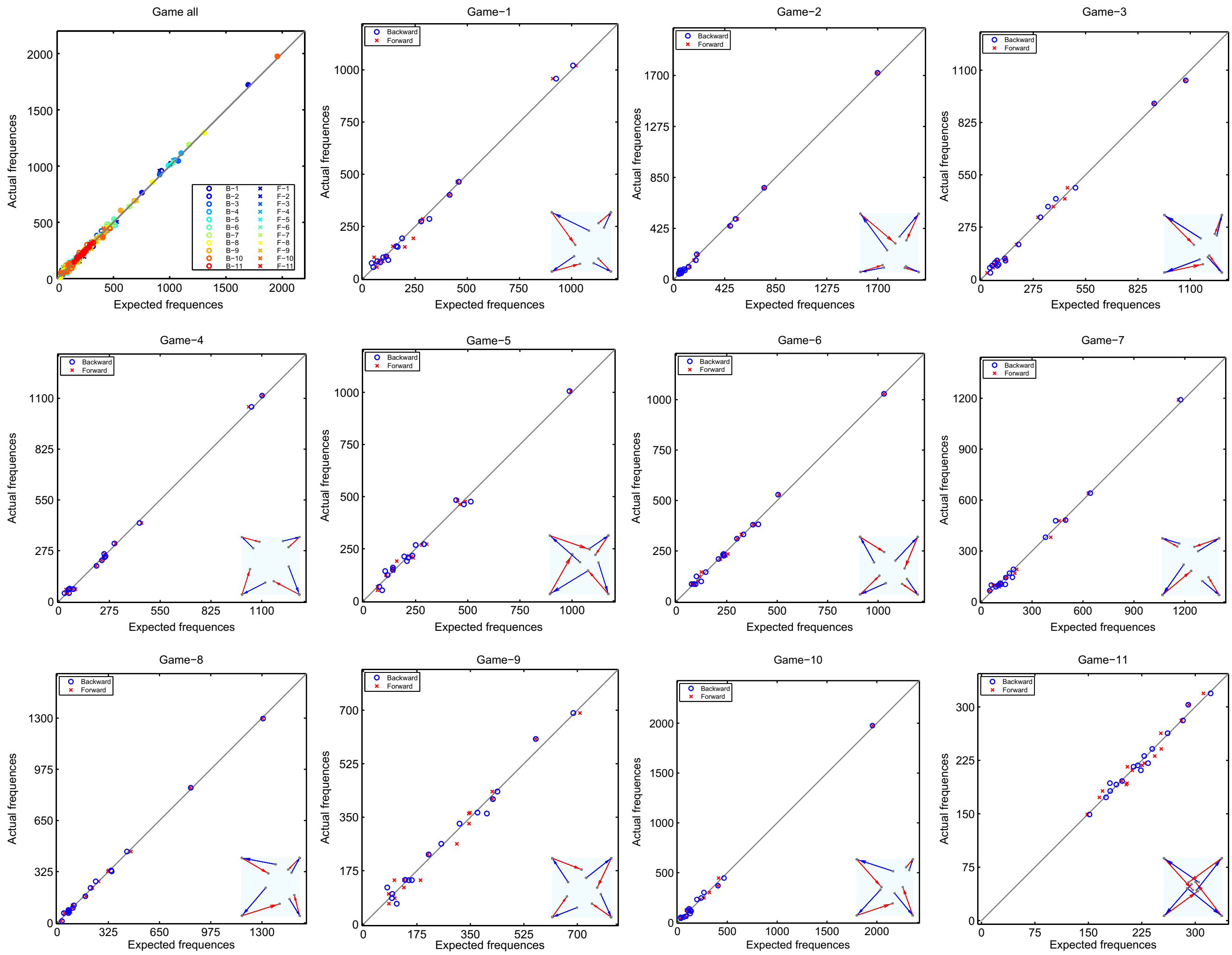

Fig. 3. Comparison of the actual transition frequencies with expected transition frequencies in the experimental games. Horizonal axis is the expected transition frequencies from MaxEnt hypothesis; meanwhile, vertical axis is the actual transition frequencies. The first figure is the results of all 11 games. Fig. 2*nd* to last is for game 1 to game 11, respectively. In the first figure, symbol B (F) in the legend indicates backward (forward). For each game, there are four social states and 16 kinds of social forward transitions and 16 kinds of backward transitions. The cycles in blue (cross in red) indicate the backward transitions (forward transitions). A dot in diagonal line means the experimental transition frequencies equal to the theoretical transition frequencies. The sub-figures in the right down corners are the schematic diagrams for aggregated backward (forward) transitions in blue arrows (in red arrows).

state can be estimated with MaxEnt, and fit the experiment data exactly.

6. Discussion and conclusion

The main result of this report is that, the social strategy transitions are not erratic but governed by MaxEnt as suggested by Jaynes [19]. This result comes from the dynamical observables in the experiments of human subject competing games [14,36].

6.1. Examples of the necessary of MaxEnt

To make the MaxEnt in social dynamics easier to understand, we provide two alternative examples in Fig. 2. The first example is in sub-figure (d), the backward transition distribution, which is consistent with the aggregated backward transition shown in (b) but not capture the experiment result in (a). In other words, in (d), the frequencies for $T_{00\leftarrow01}$, $T_{00\leftarrow00}$, $T_{00\leftarrow10}$ and $T_{00\leftarrow11}$ could be 133, 28, 0 and 271. Even though this distribution satisfied the constraint condition (b), it is far away from the experimental distribution in (a). Second example is for forward state transition in (h); without MaxEnt, a plausible distribution like the (h) in Fig. 2 does not provide efficient information of experimental dynamical observable in (e) with the constraint condition of (f). Alternatively, with MaxEnt, results in (c) and (g), by using (b) and (f) as the constraints, can recover the experimental distribution in (a) and (e), respectively.

6.2. MaxEnt in social state transition and experimental social dynamics

In this section, we explain the connection of present results to the results found in experimental social dynamics.

The social dynamics of human subject systems is an interdisciplinary field [10,17,24,37]. In this field, evolutionary game theory provides a general mathematical framework for the theoretical investigation of social dynamics and is used commonly by physicists and economics. However, this theory has rarely gained the

Table 4
Expected transitions frequencies of 11 game by MaxEnt.

T_-	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
$T_{00\leftarrow00}$	459	754	198	302	116	505	145	106	564	138	197
$T_{00\leftarrow01}$	160	62	59	79	76	124	167	100	109	117	240
$T_{00\leftarrow10}$	279	514	313	205	215	407	87	75	404	101	181
$T_{00\leftarrow11}$	97	43	93	54	141	100	100	70	78	85	219
$T_{01\leftarrow00}$	50	84	50	248	195	237	119	30	255	34	152
$T_{01\leftarrow01}$	111	50	74	1045	235	300	1176	245	373	130	213
$T_{01\leftarrow10}$	83	71	85	61	209	75	51	43	93	64	228
$T_{01\leftarrow11}$	188	43	126	254	250	96	497	346	137	238	322
$T_{10\leftarrow00}$	415	470	353	56	90	235	184	179	148	195	283
$T_{10\leftarrow01}$	69	51	129	54	104	86	60	72	95	128	261
$T_{10\leftarrow10}$	1007	1698	1076	247	444	1030	439	848	687	407	189
$T_{10\leftarrow11}$	166	184	394	234	515	380	142	343	438	265	175
$T_{11\leftarrow00}$	42	60	47	32	142	146	112	72	159	32	224
$T_{11\leftarrow01}$	121	92	87	438	481	209	379	212	314	134	180
$T_{11\leftarrow10}$	318	123	497	80	290	231	190	443	213	467	290
$T_{11\leftarrow11}$	926	192	910	1102	988	333	643	1307	424	1956	234
T_+	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	g11
$T_{00\rightarrow00}$	452	749	187	309	108	508	142	v97	563	125	197
$T_{00\rightarrow01}$	67	101	32	244	229	247	105	25	306	34	148
$T_{00\rightarrow10}$	413	461	380	50	67	256	146	183	187	223	280
$T_{00\rightarrow11}$	62	62	65	40	144	124	109	48	101	61	212
$T_{01\rightarrow00}$	143	51	84	92	67	107	198	96	83	129	252
$T_{01\rightarrow01}$	118	49	78	1029	218	303	1162	263	349	131	205
$T_{01\rightarrow10}$	95	76	88	40	144	78	70	81	83	101	252
$T_{01\rightarrow11}$	77	74	83	450	462	218	410	223	345	104	204
$T_{10\rightarrow00}$	275	531	300	202	238	382	107	85	345	88	170
$T_{10\rightarrow01}$	77	42	127	61	160	86	48	47	102	90	243
$T_{10\rightarrow10}$	1020	1695	1073	249	452	1029	460	843	708	405	203
$T_{10\rightarrow11}$	287	133	455	76	303	232	210	468	211	412	291
$T_{11\rightarrow00}$	53	45	62	55	137	114	88	81	133	44	226
$T_{11\rightarrow01}$	242	53	128	256	280	95	492	321	133	266	311
$T_{11\rightarrow10}$	201	168	441	234	488	390	113	327	422	321	165
$T_{11\rightarrow11}$	909	201	908	1104	994	322	631	1303	422	1957	229

[a] g1 indicates game 1, the rest analogize.

Table 5
Results of linear regression for T_- and T_+ of each game.

	Coef. T_- [a]	[99%C.I.] [b]		Const. [99%C.I.] [c]		Coef. T_+ [a]	[99%C.I.] [b]		Const. [99%C.I.] [c]	
g1	1.019	0.971	1.067	−24.497	13.829	1.020	0.950	1.090	−33.665	22.369
g2	1.010	0.982	1.039	−17.453	11.596	1.012	0.983	1.041	−17.993	11.337
g3	0.995	0.943	1.047	−19.863	22.717	0.997	0.952	1.041	−17.420	19.273
g4	1.009	0.981	1.037	−14.445	9.388	1.013	0.978	1.048	−18.533	11.146
g5	1.005	0.922	1.088	−31.300	28.440	1.007	0.942	1.072	−25.466	21.471
g6	1.002	0.956	1.049	−17.457	16.123	1.003	0.961	1.045	−16.216	14.447
g7	1.012	0.954	1.070	−26.656	19.858	1.017	0.970	1.065	−23.895	14.080
g8	0.997	0.968	1.026	−11.762	13.329	1.000	0.974	1.026	−11.121	11.139
g9	1.009	0.909	1.110	−36.096	30.798	1.006	0.894	1.118	−38.853	35.654
g10	1.008	0.966	1.050	−24.502	20.038	1.009	0.972	1.045	−21.852	16.953
g11	1.002	0.886	1.119	−27.159	26.109	1.011	0.848	1.175	−39.890	34.780
Total	1.007	0.995	1.019	−6.700	2.889	1.009	0.997	1.020	−7.170	2.317

[a] Liner regression coefficient.
[b] 99% Confident Interval for liner regression coefficient.
[c] 99% Confident Interval for intercept constant.

supports from laboratory experiments[1] of human subject social systems quantitatively.

Only quite recently, according to the three reports from three independent research groups, quantitative experimental testing on evolutionary dynamics is becoming possible. The three reports are the following. (1) The first is the report from Hoffman, Suetens, Nowak and Gneezy (2012) [18]. In three Rock–Paper–Scissors games, the authors compare behavior with three different symmetric matrices whose mean distances from identical Nash equilibrium (NE) are equal (unequal) in the classical (evolutionary) game theory. They find the mean distance from NE in a treatment is larger which is predicted by the replicator dynamics model in evolutionary game theory. This is the first experimental report to

[1] One point that needs to be emphasized is, most of the experiments are conducted by the social scientists in the field called as *experimental economics*. All the experiments are the incentivized laboratory experiments using human subjects. Traditional experimental testing on social dynamics mainly focus on the convergence property of the equilibrium (e.g., [6,11,12,27]). Early experiments had demonstrated the qualitative consistence between the evolutionary dynamics and laboratory social behaviors [3,13,16,29] but not the quantitative consistence. In experiment data, as pointed out [5], it is difficult to test out the dynamic patterns (e.g., cycles [5,8]) which are predicted by evolutionary game theory.

support one of the most fundamental concept — evolutionarily stable state (ESS) — in evolutionary game theory. In their report, the simplest replicator dynamics model is used as a reference. (2) The second is the report from Cason, Friedman and Hopkins (2012) [8]. Using continuous time experiments, also in Rock–Paper–Scissors games, the authors found cycles directly. More importantly, they found that the cycle amplitude, frequency and direction are consistent with standard learning models.[2] In their report [8], the logit dynamics model is used as a reference. Another important point is "time" served as controlled variable in their experiments. (3) The third is the report from Xu, Wang and Wang (2012) [32]. In the two-population random-matched 2×2 games with 12 different payoff-matrix parameters [25], the experimental frequencies are found to be linear positive related to the theoretical frequencies significant (>5σ). The payoff-normalized replicator dynamics model is used as the reference. Together with the observed cyclic velocity vector field pattern in experiments [33], evidences from 2×2 games support the evolutionary game theory as well. To test the evolutionary dynamics in laboratory human subject social systems, these are the three experiments which are reported recently.

Notice that, all the three groups use accumulated observable (macro observable) to describe the social dynamics behaviors. In this letter, the macro observables which are used as the constraint conditions (e.g., the mean aggregated forward transition) are also macro observable. In this letter, we show that MaxEnt can provide more dynamics information (micro observable, the state-to-state transits) from the limited accumulated observable (macro observable) in the experiments.

In words, present report might provide a paradigm — the micro and macro dynamical observables could be linked by MaxEnt in social dynamical processes in experiments. We suggest the results reported in this letter could be replicated in more general conditions in the experiments of human subject social dynamics.

6.3. MaxEnt as a link between nature and social science

In economics, MaxEnt approach has gained its widely applications, e.g., in market equilibrium [2,28], in wealth and income distribution [9,31], in firm growth rates [1] in behavior modeling [30]. Theoretical interpreting or modeling of the distributions of social outcomes with MaxEnt is growing.

However, to the best of our knowledge, the dynamic behavior (both of the backward and forward transitions) obeys MaxEnt — this point has never been empirically presented. Our finding of the MaxEnt in dynamic behaviors in the experimental data can be an encouraging information for investigating the potential self-consistence of social outcome — both in static and dynamic performance.

MaxEnt, as a technique, can be used to predict the geographic distribution of any spatial phenomena, including plants and animals [22]. In the game theory condition, the spatial phenomena of social behavior is the phenomena in strategy space, at the same time, the absence or appearance of strategy is relative to the absence or appearance of species. This picture has been well built [24] to unify the evolutionary theorems in biology science and social science. Our findings of the social behavior fitting MaxEnt, both in dynamic respect in this report and in static respect in [36], suggest that human subject social systems and natural systems could have wider common backgrounds.

For the future investigations, several points need to be considered. As we have shown in static [36] and in one step ($x_{t\pm1}$) dynamics social behaviors obey MaxEnt, does any step transitions always obey MaxEnt? What is the bound of the MaxEnt in social interaction systems?

One can notice that, in the 11 games, all the social environments are different (for the payoff matrix is different), all the mixed strategy Nash equilibrium are different, however, all the social transitions obeying MexEnt are indifferent.

[2] This findings of cycles in Rock–Paper–Scissors games [8] are supported by the discrete time experiments of three different parameters of Rock–Paper–Scissors games [34,35] from an independent research group.

6.4. Conclusion

By employing experimental economics data, we test the MaxEnt hypothesis in social transitions. In the experimental constant sum two-person 2×2 games, the results show that, not only the static social state distributions obey MaxEnt [36], the distributions of the social state transitions also fit MaxEnt. This finding suggests that MaxEnt can also be an approach for the social dynamics.

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