Experimental Economics, 2:41–57 (1999)
© 1999 Economic Science Association

Continuous-Time Strategy Selection in Linear Population Games

SIEGFRIED K. BERNINGHAUS, KARL-MARTIN EHRHART AND CLAUDIA KESER
Institute for Statistics & Mathematical Economics, University of Karlsruhe

Abstract

In an experimental evolutionary game framework we investigate whether subjects end up in a socially efficient state. We examine two games, a game where the socially efficient state is also an equilibrium and a game which has no equilibrium in pure strategies at all. Furthermore, we distinguish between a situation in which the subjects are completely informed about the payoff function and a situation in which they are incompletely informed. We observe that subjects spend the greater part of the time at or near the efficient state. If the efficient state is an equilibrium, they spend more time there than otherwise. Furthermore, incomplete information increases the time spent at the efficient state.

Keywords: evolutionary dynamics, incomplete information, continuous-time games

1. Introduction

In this paper we report on experiments which were designed to examine *evolutionary dynamics* in a simple game theoretical framework. In our framework the evolutionary aspect is generated by two features. First, we consider a "population game" where each player's payoff depends on the distribution of strategies in a finite population of players. Second, we allow for strategy selection in *continuous time*. This distinguishes our experiment from most previous experiments, which are in discrete time. In discrete time, one would have to run hundreds of repetitions in order to justify the idea of evolution. The only experiments in (almost) continuous time which we know of were previously run by Selten and Berg (1970), Berninghaus and Ehrhart (1998), Millner et al. (1990), and Ehrhart (1997).

The major aim of our paper is to investigate under which conditions subjects reach an equilibrium strategy distribution in an experimental framework. There exists an extensive theoretical literature on game dynamics, where it has been demonstrated that even fairly bounded rational players may end up in a Nash equilibrium (see, for example, Milgrom and Roberts, 1991; Kalai and Lehrer, 1993; or Fudenberg and Kreps, 1988). Experiments which have been run on this topic were mostly designed to investigate *equilibrium selection* in symmetric coordination games (see, for example, van Huyck et al., 1990) or to examine behavior in cooperation games (Ledyard, 1995). In our experiments we compare the strategy choice in a population game with a unique equilibrium which is socially efficient to the strategy choice in a game lacking an equilibrium. In the population game without an equilibrium, we investigate whether socially efficient distributions are focal points in strategy selection. Furthermore, we examine the role of information in our experiments. In many real life games we are not informed, for example, about our payoff functions or about the distribution of strategies chosen in the population of players. Often, we only know the sequence of realized monetary payoffs, but we do not know our payoff *function*. What are the principles guiding strategy selection in such a situation with incomplete information? Can we expect less regular behavior in strategy choice when players are less informed about their environment?

We examine the behavior of subjects in games with nine players each. Each player is in the same decision situation and can choose among three strategies. We consider two different games. Game I has an asymmetric equilibrium which results from a unique distribution of the players' strategy choices. This equilibrium is also Pareto-efficient. Game II has the same Pareto-efficient solution, which, however, is not an equilibrium. It has no equilibrium in pure strategies at all. In game I, subjects spend about 75% of the time at or near the Pareto-efficient state, while they spend significantly less time there in game II (66%). Furthermore, in a variant of game I where subjects are not informed of the payoff function, they spend even more time at or near the Pareto-efficient state than under complete information (85%). Here, an *invisible hand* seems to be at work, leading to a Pareto-efficient outcome. This result is in line with Smith's (1994) observation that on markets less information can be better than more information. Interestingly, in all games the observed percentage of time spent at or near the efficient state is more or less constant over time. In this sense we observe no evolution of the strategy distribution. The same result holds for the observed average payoff per minute.

Finally, we relate a player's payoff to the frequency with which he switches strategies during the course of the game. We can show that it does not pay to be very active in changing strategies. In game I there is a negative correlation between the frequency of a player's switching strategies and the resulting payoff. We find an interesting analogue to our result in financial markets. It has been observed that in recent years big fund companies which changed their portfolios more often performed worse than more passive companies pursuing a simple index strategy in their portfolio choice.

2. The experiment

2.1. General description of the population game

We consider a simple normal form game with a finite number of players N. Each player has to choose among 3 strategies labeled σ_k ($k = 1, 2, 3$). The payoff of each strategy $\sigma_k \in \Sigma$ depends on the *distribution of strategies* in the population. A *strategy distribution* is given by a triple $n = (n_1, n_2, n_3)$ of natural numbers, where n_k denotes the number of players choosing strategy σ_k. The set of all admissible strategy distributions is then given by the set

$$\mathcal{N} := \left\{ n = (n_1, n_2, n_3) \in \mathbb{N}_0^3 \;\middle|\; \sum_k n_k = N \right\}.$$

Furthermore, we stipulate that players can select only pure strategies.

The payoff $\pi_k(n)$ of a player selecting strategy k depends on the prevailing strategy configuration n. It is represented as follows

$$\pi_1(n) = an_1 + bn_2 + cn_3 \quad (1)$$

$$\pi_2(n) = an_2 + bn_3 + cn_1 \quad (2)$$

$$\pi_3(n) = an_3 + bn_1 + cn_2 \quad (3)$$

with arbitrary real numbers a, b, c.

Although our payoff function seems to be quite specific, we believe that it is applicable to many economic decision situations. As a first example, consider the models on equilibrium price or wage dispersions in oligopolistic markets with uninformed consumers (see Carlson and McAfee, 1983; Berninghaus, 1984), where the pricing decisions of the oligopolistic firms usually result in a nondegenerate distribution of market prices. Consumers, in these models, are supposed to know only this distribution of prices and employ a sequential price search rule, in order to determine at which store an item is purchased. One can show that the expected profit of a single firm depends on the overall distribution of market prices. Each firm has to take account of the effect of its own price decision on the price distribution. It is well known from the literature that under specific assumptions regarding the consumers' search costs, a uniform equilibrium price dispersion may result. Our experimental results (see Section 3 below) show a strong tendency on the part of the subjects to disperse their strategies uniformly over the strategy set.

Another application concerns markets without sunk costs.[1] As a concrete example, let us consider a number of airlines who plan to use one of three airports for landing. If changing the airport is possible without any significant switching costs, then the airlines' decision problems may be described in principle by our simple strategic decision model. Here, it makes sense to assume that a single airline's payoff depends only on the distribution of airlines over airports and not on the individual strategy configuration.

A similar decision problem can arise in big fund companies on financial markets. Managers in these companies have to choose particular portfolios of securities. They may alter their portfolio at any time they want. The profit of a particular portfolio crucially depends on the distribution of portfolios over the remaining fund companies. In the final part of our paper we will describe an interesting parallel between our experimental results on the impact of the intensity of strategy switching and some specific empirical findings for large fund companies.

Finally, our model also has aspects in common with so-called congestion games (see Rosenthal, 1973). In these games, selecting a pure strategy means utilizing a subset of a pool of primary resources. Since all players are allowed to use the same resources, it is easy to see that the players may run into a congestion problem. In such a model, it is crucial for a single player to know the *number* of other players choosing the same resource, but it is not as important to know which individual player uses a particular resource.

2.2. *Equilibrium solutions*

Since individual strategy choices do not play a significant role in our framework, we formulate our equilibrium concept for strategy distributions exclusively. An *equilibrium strategy distribution* n^* is defined in the Nash sense as follows.

Definition 1. A strategy distribution $n^* = (n_1^*, n_2^*, n_3^*)$ is an equilibrium if the following inequalities hold:

$$\forall j : [\forall k : \pi_j(n^*) \geq \pi_k(n^*_{j-k})], \tag{4}$$

where n^*_{j-k} denotes the strategy distribution which results from n^* by increasing n_k^* by 1 and diminishing n_j^* by 1.

This definition simply says that for each representative player choosing strategy σ_j it is not profitable to deviate to any strategy $\sigma_k \neq \sigma_j$. In calculating his payoffs, a deviating player has to take into account the altered strategy distribution. Therefore, an equilibrium distribution in our sense is only an aggregate description of ordinary Nash equilibrium strategy configurations. An *equilibrium distribution* $n^* \in \mathbb{N}_0^3$ may be generated by several individual Nash *equilibrium configurations* $(\sigma_{1_{k_1}}, \ldots, \sigma_{N_{k_n}}) \in \Sigma^N$.

We are able to entirely characterize the equilibrium strategy distributions under the assumption that the relation

$$n_1 + n_2 + n_3 = 3k \quad \text{with } k \in \mathbb{N}_+ \tag{5}$$

holds.

Proposition 1. *For $a \neq b \vee a \neq c$ the population game has two types of equilibria in pure strategies:*
1. $(\frac{N}{3}, \frac{N}{3}, \frac{N}{3})$ *for $a \leq b \wedge a \leq c$ and at least one inequality is strict.*
2. $(N, 0, 0)$, $(0, N, 0)$, *and* $(0, 0, N)$ *for $a \geq b \wedge a \geq c$ and at least one inequality is strict.*

Proof: See Appendix A. □

For $a = b = c$ each configuration $n \in \mathcal{N}$ is an equilibrium. Each player's payoff is equal to aN, whatever the configuration.

2.3. *Experimental implementation*

In our experiments we consider two specific examples, called game I and game II. The number of players is set as $N = 9$ (such that condition (5) is satisfied). The set of admissible strategy distributions is thus given by the finite set

$$\mathcal{N} = \{(n_1, n_2, n_3) \in \mathbb{N}_0^3 \,|\, n_1 + n_2 + n_3 = 9\}.$$

Game I and game II are induced by two different instances of the payoff parameters a, b, and c. For game I, we set

$$a = 2, \quad b = 4, \quad c = 6,$$

and for game II, we set

$$a = 3, \quad b = 2, \quad c = 7.$$

By straightforward application of the results in Proposition 1, we see that game I has a unique distribution equilibrium $n^* = (3, 3, 3)$. The equilibrium payoff in game I is the same for each of the strategies. It is equal to $\pi_1(n^*) = \pi_2(n^*) = \pi_3(n^*) = 36$. Game II has no equilibrium (in pure strategies) at all. One can show that in both games a unique *completely mixed strategy equilibrium* exists in which each player chooses each pure strategy with a probability of 1/3 since only by mixing pure strategies in this way the payoffs of all three strategies are equalized.

An equilibrium in our game is a stable distribution of strategies, but we cannot guarantee that this distribution is desirable from a social point of view. Therefore, we round out our analysis with a brief discussion of welfare properties of the strategy distributions in games I and II. To be more precise, we regard a distribution $n = (n_1, n_2, n_3)$ as *socially efficient* when it maximizes $f(n_1, n_2, n_3) := n_1\pi_1(n) + n_2\pi_2(n) + n_3\pi_3(n)$, the overall total payoff in the population, over the set of all admissible distributions $\mathcal{N}$. It can be shown (see Appendix B) that $(3, 3, 3)$ is the unique maximizer of $f(\cdot)$ in both games. Hence, in game I the equilibrium is socially efficient. Although game II has no equilibrium, $(3, 3, 3)$ is the unique socially efficient distribution of strategies in the population.

2.4. *Experimental design*

The game was presented to the subjects as a symmetric 9-player game in which each player may choose among strategies X, Y, and Z.[2] A player's payoff depends on his choice and the total number of players who choose X, the number of players who choose Y, and the number of players who choose Z, including the player's own choice. The payoff is determined as presented in Table 1 (game I) and Table 2 (game II). Note that x, y, and z denote the total number of players who choose strategy X, Y, and Z respectively, and that $x + y + z = 9$.

Table 1. Payoff table for game I.

Your choice	Your payoff
X	$2x + 4y + 6z$
Y	$6x + 2y + 4z$
Z	$4x + 6y + 2z$

Table 2. Payoff table for game II.

Your choice	Your payoff
X	$3x + 2y + 7z$
Y	$7x + 3y + 2z$
Z	$2x + 7y + 3z$

Repetition of the game in continuous time. The game lasted 15 min. It started when all subjects had made their first decisions. Thereafter, subjects could change their strategies at any time. The current payoff was computed every $\frac{1}{10}$ of a second and "integrated" over time. Some information was presented graphically on the subjects' computer screens throughout the game. This included the number of subjects choosing X, Y, and Z; plus the current payoff for someone playing each of strategies X, Y, and Z. Furthermore, the subject's own current decision, the cumulative payoff, and the elapsed time were indicated on each screen during the entire course of the experiment.

Underlying our method of experimentation is a theoretical conviction. We believe that continuous time experimentation is important in the social sciences. In natural social systems there is not often something as a global clock, which causes all members of the system to simultaneously update their strategies.

We know from the theory of dynamical systems (see, for example, Huberman and Glance, 1993) that the dynamics of strategy selection processes in an asynchronous world is completely different from the dynamics in a synchronous world. Their results show that it may be misleading to attempt to draw valid conclusions about real-world systems from models of synchronous strategy adaptation.

Organization of the experiments. We ran the experiments at the University of Karlsruhe, Germany. Subjects were students from various disciplines. Through notices on billboards around the university campus, we continuously recruit students for a pool of potential subjects. Out of this subject pool we randomly selected students for participation in these experiments. As hinted above, the experiments were computerized. Each subject was seated at a computer terminal which was separated from the other subjects' terminals by wooden screens. The subjects received written instructions, which were also read aloud by a research assistant. Before the experiment started, each subject had to answer at his computer terminal some questions with respect to the instructions. Only after all subjects had given the right answers to all questions was the experiment started. No communication other than through the decision making, was permitted. After the experiment, subjects were paid in cash according to their payoffs in the game.

Treatments. In each of the experiments, we examined either game I or game II. For each game, we examined both a complete information and an incomplete information situation. In the complete information experiments, subjects were informed of the rules of the game, including the payoff function, while in the incomplete information experiments subjects were not informed of the payoff function. Note, however, that in all experiments subjects were informed at all times what proportion of subjects was playing strategies X, Y, and Z and what the current payoff was for someone playing strategy X, Y, or Z.

We organized sessions with 18 subjects each. Thus, subjects could not identify which of the other subjects belonged to their game population. We thus obtained two independent observations in a session. In total, we have 4 independent observations of each treatment (game I with complete information, game I with incomplete information, game II with complete information, game II with incomplete information).

3. Aggregate experimental results

In this section we present some aggregate results with respect to strategy distributions, number of strategy changes, and payoffs.

3.1. *Strategy distributions*

In the experiment, we recorded every instance when any of the subjects made a new choice. We registered the new choice and the time elapsed since the last recording.

We report in Table 3 on the percentage of time the players' strategy choices were X, Y or Z.

We see that there is no strict preference for any strategy in the population.

The results in Table 3 concern single strategy choices. As a further step we investigate distributions of strategies. Table 4, reports for each population in each treatment, the total time over the whole game during which the nine subjects are in a Pareto-efficient state, i.e., in a situation where each of the three strategies is chosen by three subjects. Table 4, also reports the total time over the whole game during which each population is in an almost efficient situation. We consider all configurations to be almost efficient situations, in which, one strategy is chosen by three subjects while another strategy is chosen by four subjects and the remaining strategy by two subjects. There are six configurations of this kind.

With complete information, subjects spend about 75% of the time in the efficient or an almost efficient state in game I, while in game II subjects spend about 66% of the time in such states. The difference between game I and game II is statistically significant at the 5% significance level (one-sided U-test). We conclude that in game I, where the efficient state also is an equilibrium, subjects tend to spend more time in or near the efficient state than in game II, where the efficient state is not an equilibrium.

This is also true for the experiments with incomplete information. In game I, subjects spend about 85% of the time in the efficient or an almost efficient state while in game II subjects spend about 65% of the time in such states. The difference between game I and game II is statistically significant at the 5% significance level (two-sided U-test).

Table 3. Percentage of time spent choosing X, Y, Z.

		Strategies		
Game	Information	X	Y	Z
I	Complete	33.6	32.9	33.6
	Incomplete	33.9	32.8	33.3
II	Complete	33.9	33.0	33.2
	Incomplete	34.3	32.8	32.9
	Average	33.9	32.9	33.2

Table 4. Percentage of time spent in the efficient situation, in an almost efficient situation, and in all other situations.

	Complete information				Incomplete information			
		Percentage of time spent in				Percentage of time spent in		
Game	Pop.	Efficient state	Eff. + almost efficient state	Other states	Pop.	Efficient state	Eff. + almost efficient state	Other states
I	1	21.2	75.4	24.6	1	48.3	89.3	10.7
	2	25.8	78.1	21.9	2	28.3	82.7	17.3
	3	26.1	74.9	25.1	3	34.7	83.4	16.6
	4	21.2	70.1	29.9	4	33.7	83.3	16.7
	Average	23.6	74.6	25.4	Average	36.3	84.7	15.3
II	1	14.7	63.5	36.5	1	19.9	71.5	28.5
	2	15.2	58.1	41.1	2	12.7	58.6	41.4
	3	16.4	63.7	36.3	3	14.4	71.9	38.1
	4	18.1	80.3	29.7	4	15.1	58.7	41.3
	Average	16.1	66.4	35.9	Average	15.5	65.2	37.3

When we restrict ourselves to the consideration of the time spent in the efficient state, we similarly observe that in game I subjects tend to spend more time in the efficient state than in game II (two-sided U-tests, 5% significance level). Apart from the difference between game I and game II, we observe for game I a significant difference between the complete and the incomplete information situation. In the incomplete information situation, subjects spend significantly more time in the efficient (or an almost efficient) state than in the complete information situation (two-sided U-test, 5% significance level). This is also illustrated in figure 1 below. It shows for each of the 15 min of play the percentage of time spent in the

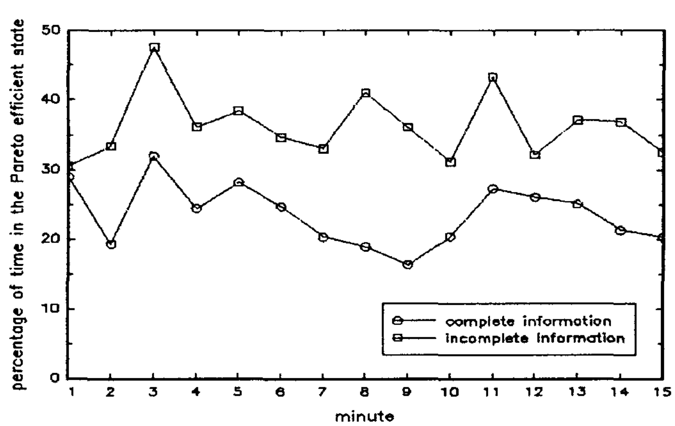

Figure 1. Percentage of time per minute spent in the Pareto-efficient state (game I).

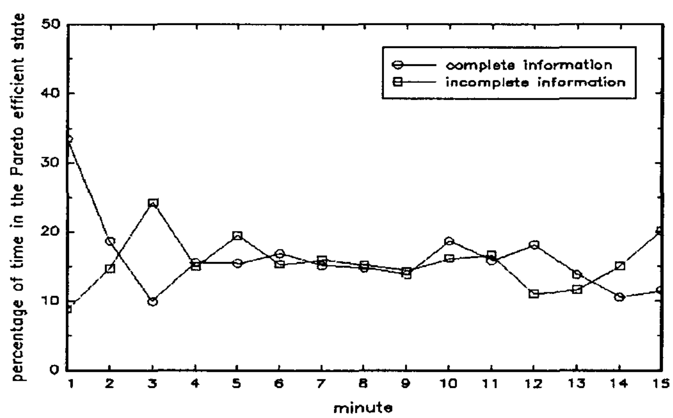

Figure 2. Percentage of time per minute spent in the Pareto efficient state (game II).

efficient state over the 4 populations in game I with complete information and over the 4 populations in game I with incomplete information.

For game II we do not observe a significant difference between the complete and the incomplete information situation. This is also illustrated in figure 2. It shows for each of the 15 min of play the percentage of time spent in the efficient state over the 4 populations in game II with complete information and over the 4 populations in game II with incomplete information.

Figures 1 and 2 also convey that the time per minute spent in the efficient state is about the same over the whole experiment. We do not observe an increase over time, as one might have expected.

3.2. *Number of strategy changes*

Table 5 reports for each population, in each treatment, the average number of changes per player per minute. In game I, we observe significantly more changes in the case of complete information than in the case of incomplete information (two-sided U-test, 5% significance level). This is also illustrated in figure 3, which shows the average number of changes per player in each of the 15 min.

Similarly, we observe for game II more changes on average in the case of complete information than in the case of incomplete information. This is illustrated in figure 4, which shows the average number of changes per player in each of the 15 min in game II. However, the difference is not significant if we require a significance level of 5% (one-sided U-test).[3] Note also that between game I and game II, we observe no significant difference in the number of changes. Interestingly, in all treatments the number of changes increases over time, as can be seen from figures 3 and 4 below.

3.3. *Payoffs*

Finally, we present some aggregate results on the average payoff per minute, which are illustrated in figures 5 and 6 for game I and II, respectively.

Table 5. Average number of changes per player and minute.

	Complete information		Incomplete information	
Game	Population	# Changes	Population	# Changes
I	1	11.9	1	5.7
	2	10.6	2	10.6
	3	13.7	3	7.5
	4	13.6	4	7.5
	Average	12.5	Average	7.8
II	1	12.9	1	7.8
	2	14.1	2	11.7
	3	11.0	3	10.2
	4	9.2	4	9.3
	Average	11.8	Average	9.7

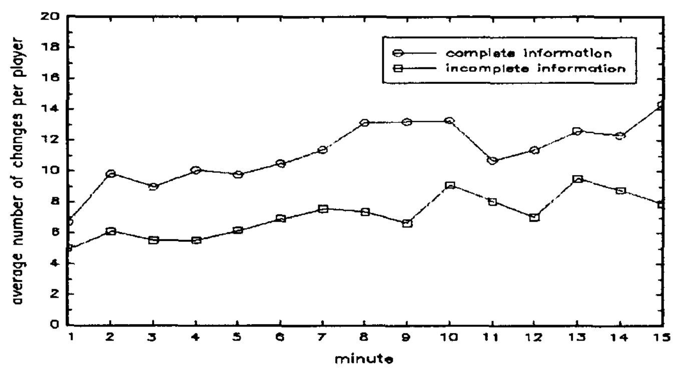

Figure 3. Average number of changes per player and minute (game I).

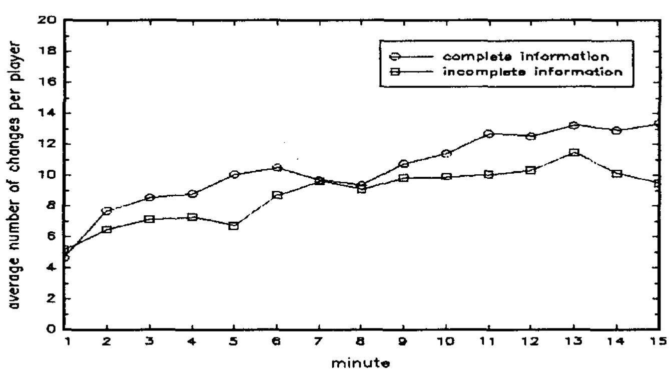

Figure 4. Average number of changes per player and minute (game II).

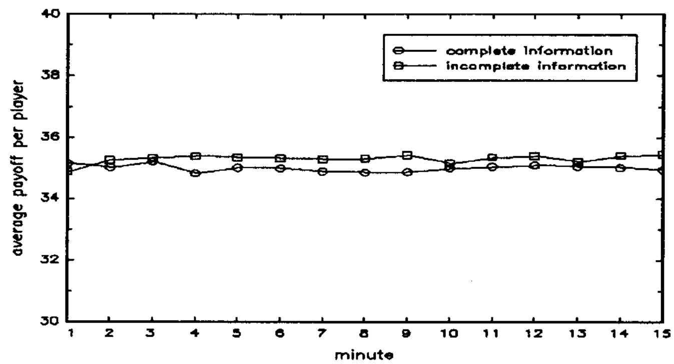

Figure 5. Average payoff per player per minute (game I).

Interestingly, the average payoff per player per minute shows almost no time evolution in all treatments irrespective of the information on payoff functions given to the players. This contrasts sharply to the players' increasing activity levels shown in figures 3 and 4 above. We will elaborate on this phenomenon in the next section by evaluating individual data.

4. Individual experimental results

We display the relationship between a player's activity and his average payoff in more detail in Table 6.

Considering the average number of changes per minute (seventh column in Table 6), we find support for our previous results that completely informed players are more active than incompletely informed players. Three other results reported in Table 6 deserve more detailed comments. First, comparing the standard deviations of the average payoff and of

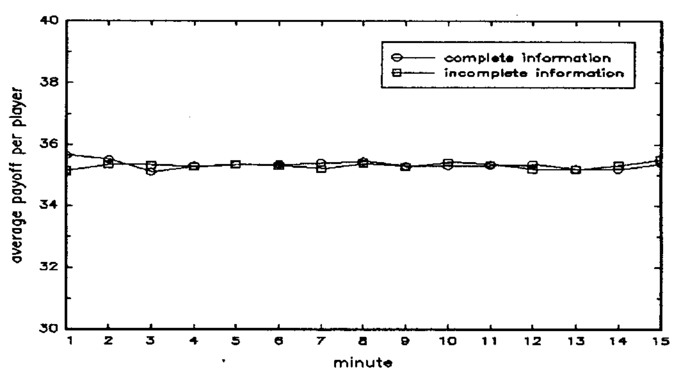

Figure 6. Average payoff per player per minute (game II).

Table 6. Payoff and players' activities.

		Average payoff per minute per player				Average number of changes per minute per player				
Game	Information	Avg.	Min	Max	Std.-dev.	Avg.	Min	Max	Std.-dev.	Spearman r.c.c. (p-value)
I	Complete	35.00	34.11	35.70	0.35	12.46	0.33	37.00	7.27	−0.82 (0.000)
	Incomplete	35.30	34.31	36.11	0.43	7.80	0.00	22.67	6.48	−0.94 (0.000)
II	Complete	35.34	34.25	37.21	0.51	11.79	0.93	28.93	7.33	0.19 (0.129)
	Incomplete	35.30	34.75	36.05	0.35	9.74	2.33	24.00	4.59	0.20 (0.117)

the average number of changes per minute, we see only slight deviations from the average payoff but extremely large deviations from the average number of strategy changes over the groups in the treatments. The maximum and minimum individual payoff per minute (fourth and fifth columns in Table 6) do not differ significantly from each other in any of the 4 treatments. However, we find extremely large differences between the maximum and minimum of the individual strategy changes per minute (eighth and nineth columns in Table 6).

Second, Spearman's rank correlation coefficient between the payoff per minute and the number of changes per minute (final column of Table 6) teaches us that there is a significant negative correlation between a player's activity and his payoff in game I. For game II we do not obtain a significant result.

Third, we see from the third column of Table 6 that the average payoff per player per minute does not vary significantly among the four treatments. It is slightly below the equilibrium or socially efficient payoff which is equal to $\pi_k(n^*) = 36$. Moreover, we already know from figures 5 and 6 above that the average payoff shows almost no time evolution in all four treatments.

When we look at the experimental results and consider the individual payoff change at the moment when a decision is recorded, we observe increases in the players' payoff about 54% of the time in game I (both with complete and incomplete information). In game II, the proportion of increases is about 65% (both with complete and incomplete information).[4] When we investigate the relation between decision changes and payoff changes, we face the following problem. Due to the experimentation in continuous time, clear reference points are missing. Obviously, subjects are not able to react instantaneously to what they observe on their screens; they need some time to react. Therefore, we do not only consider the payoff change relative to the configuration which exists at the moment of the change; we also consider the configurations which existed before the decision change. In both games the average proportion of payoff increases is highest relative to configurations which existed between $\frac{7}{10}$ and $\frac{6}{10}$ of a second backwards. If we take $\frac{1}{10}$ of a second "computer delay time" into account, we can conclude that the subjects' average reaction time is about half a second.

We observe that in all populations in game I there are subjects who deviate from the Pareto-efficient equilibrium. To deviate seems to be "irrational", since subjects hurt themselves. Reaction time can hardly be responsible for this, because we observe deviations even if an equilibrium has existed for a longer time period. There may be other explanations for this kind of behavior. First, subjects might want to experiment with their strategies and payoffs. Second, Berninghaus and Ehrhart (1998) show that spitefulness may be a motive to deviate from a Pareto-efficient equilibrium, since a deviating subject also induces payoff decreases for other subjects. Third, deviating from the equilibrium usually induces a sequence of decision changes in the population. Subjects who deviate may believe that during a sequence of nonequilibrium states they have a chance of earning more money than they lose by deviating from the equilibrium.

A unilateral deviation from the equilibrium induces either a payoff decrease of -2 ExCU/minute or a decrease of -4 ExCU/minute for the deviating player.[5] Do the subjects who deviate from the equilibrium prefer a specific decision? Do they choose the strategy which induces the smaller payoff decrease? Analyzing the experimental data, we do not find support for these hypotheses. In changing actions, subjects choose the two remaining alternatives with approximately equal frequency, regardless of their initial choices X, Y, or Z.

5. Summary

We have now seen that subjects in our population game spend most of the time at or near the efficient state. They tend to spend more time there if the efficient solution is also an equilibrium. Interestingly, in the case of an equilibrium a lack of information regarding the payoff function increases the time spent at or near the efficient state even more. We do not yet have an explanation for this phenomenon. It is, however, important to note that in both games, subjects change their decisions in the case of complete information more often than in the case of incomplete information. Subjects who are more active in switching strategies do not perform better than passive players with respect to their payoffs. This result has an important impact on the payoffs of completely informed and incompletely informed players, who differ significantly with respect to their degree of activity in switching strategies.

Our surprising result of negative correlation between strategy switching and payoff (see Table 6) has an interesting empirical counterpart on financial markets. It is a well-supported belief among financial market analysts (source Dow Jones Interactive Publication Library: Financial Times 01/05/98, Wall Street Journal 01/17/97) that fund companies with active portfolio managers performed significantly worse than so-called index fund companies. In 1997, for example, the average equity fund gained 19.47% compared with 22.95% for index funds. We do not intend to explain complicated financial market transactions by our simple model. But, having the market interpretation of our model in mind, we do not believe that the similarity of our results with real market phenomena is purely random.

Appendix A

Proof of Proposition 1

We consider a configuration $(n_1, n_2, n_3) \in \mathcal{N}$. If a player unilaterally changes from strategy i to j, let $\pi_{i-j} = \pi_j(n_{i-j})$ denote the player's payoff when adopting strategy j. For $n_i > 0$

we obtain:

$$\pi_{1-2} = a(n_2 + 1) + bn_3 + c(n_1 - 1) \tag{6}$$

$$\pi_{1-3} = a(n_3 + 1) + b(n_1 - 1) + cn_2 \tag{7}$$

$$\pi_{2-1} = a(n_1 + 1) + b(n_2 - 1) + cn_3 \tag{8}$$

$$\pi_{2-3} = a(n_3 + 1) + bn_1 + c(n_2 - 1) \tag{9}$$

$$\pi_{3-1} = a(n_1 + 1) + bn_2 + c(n_3 - 1) \tag{10}$$

$$\pi_{3-2} = a(n_2 + 1) + b(n_3 - 1) + cn_1 \tag{11}$$

Furthermore, to simplify notation we define

$$n_{ij} = n_i - n_j \quad \text{for } i, j = 1, 2, 3. \tag{12}$$

These differences have the following properties:

$$n_{ij} = -n_{ji} \tag{13}$$

$$n_{ii} = 0 \tag{14}$$

$$n_{ij} = n_{ik} + n_{kj} \tag{15}$$

If a player unilaterally changes from strategy i to j let $\Delta\pi_{ij} = \pi_{i-j} - \pi_i$ denote the difference between the payoff the player receives by adopting strategy j and the payoff the player previously received by choosing strategy i. For $n_i > 0$ we obtain:

$$\Delta\pi_{12} = a(n_{21} + 1) + bn_{32} + c(n_{13} - 1) \tag{16}$$

$$\Delta\pi_{13} = a(n_{31} + 1) + b(n_{12} - 1) + cn_{23} \tag{17}$$

$$\Delta\pi_{21} = a(n_{12} + 1) + b(n_{23} - 1) + cn_{31} \tag{18}$$

$$\Delta\pi_{23} = a(n_{32} + 1) + bn_{13} + c(n_{21} - 1) \tag{19}$$

$$\Delta\pi_{31} = a(n_{13} + 1) + bn_{21} + c(n_{32} - 1) \tag{20}$$

$$\Delta\pi_{32} = a(n_{23} + 1) + b(n_{31} - 1) + cn_{12} \tag{21}$$

The necessary and sufficient conditions for a strategy configuration (n_1, n_2, n_3) with $n_i > 0$ to be a Nash equilibrium are

$$\Delta\pi_{ij} \leq 0 \; \forall i \neq j. \tag{22}$$

To characterize all possible Nash equilibria we distinguish between a symmetric configuration and asymmetric ones. Furthermore, for asymmetric configurations we have to consider three different cases with respect to the relation among a, b, and c.

First, there might be a symmetric equilibrium configuration $(\frac{N}{3}, \frac{N}{3}, \frac{N}{3})$.

In that case it follows from (16)–(22) that $\Delta\pi_{ij} = a - b \leqq 0$ and $\Delta\pi_{ij} = a - c \leqq 0$ have to be satisfied for all $i \neq j$. Therefore, $a \leq b \wedge a \leq c$ is a necessary and sufficient condition for $(\frac{N}{3}, \frac{N}{3}, \frac{N}{3})$ being a Nash equilibrium.

Second, there might be asymmetric equilibrium configurations where $(n_1, n_2, n_3) \neq (\frac{N}{3}, \frac{N}{3}, \frac{N}{3})$. Because of the specific symmetry in the game we can assume $n_1 = \max\{n_i\}_i$ without loss of generality. From (5) we get

$$n_1 \geq \min\{n_i\}_i + 2. \tag{23}$$

1. We consider the case $c \geq b \geq a$ in which at least one inequality is strict. Because of the symmetry of the game, the case $b \geq c \geq a$ is equivalent.

   (a) Let $n_3 = \min\{n_i\}_i$. We consider a player changing from strategy 1 to strategy 3. With (17), $c \geq b, n_{23} \geq 0$, and $n_{13} = n_{12} + n_{23} \geq 2$, we get

   $$\begin{aligned}\Delta\pi_{13} &\geq a(n_{31} + 1) + b(n_{13} - 1) \\ &= (b - a)(n_{13} - 1) \geq 0.\end{aligned}$$

   For $c > b$ the first inequality is strict and for $b > a$ the second inequality is strict. This implies $\Delta\pi_{13} > 0$, which contradicts (22).

   (b) Let $n_2 = \min\{n_i\}_i$ and $n_3 > n_2$. We consider two possibilities. Either a player changes from strategy 1 to 2 or another player changes from strategy 3 to 2. With (16), (21), $c \geq b, n_{12} + n_{13} \geq 1$, (13), and $n_{12} + n_{32} \geq 3$, we get

   $$\begin{aligned}\Delta\pi_{12} + \Delta\pi_{32} &= a(n_{21} + n_{23} + 2) + b(n_{31} + n_{32} - 1) + c(n_{12} + n_{13} - 1) \\ &\geq a(n_{21} + n_{23} + 2) + b(n_{13} + n_{32} + n_{31} + n_{12} - 2) \\ &= (b - a)(n_{12} + n_{32} - 2) \geq 0.\end{aligned}$$

   For $c > b$ the first inequality is strict and for $b > a$ the second inequality is strict. This implies $\Delta\pi_{12} + \Delta\pi_{32} > 0$, which contradicts (22) because at least one of the two players can raise his payoff by unilaterally changing his strategy.

2. We consider the case $c > a \geq b$ in which at least one inequality is strict. Because of the symmetry of the game, the case $b > a \geq c$ is equivalent.

   (a) Let $n_3 = \min\{n_i\}_i$. We consider a player changing from strategy 1 to 2. With (16), $c > a, n_{13} \geq 2$, and $n_{23} \geq 0$ we get

   $$\begin{aligned}\Delta\pi_{12} &> a(n_{21} + n_{13}) + bn_{32} \\ &= (a - b)n_{23} \geq 0.\end{aligned}$$

   This implies $\Delta\pi_{12} > 0$, which contradicts (22).

(b) Let $n_2 = \min\{n_i\}_i$ and $n_3 > n_2$. We consider a player changing from strategy 3 to 2. With (21), $c > a$, $n_{12} \geq 2$, and $n_{13} \geq 0$, we get

$$\begin{aligned}\Delta\pi_{32} &> a(n_{12} + n_{23} + 1) + bn_{31-1}\\ &= (a - b)(n_{13} + 1) \geq 0.\end{aligned}$$

This implies $\Delta\pi_{32} > 0$, which contradicts (22).

3. We consider the case $a \geq c \geq b$ in which at least one inequality is strict. Because of the symmetry of the game, the case $a \geq b \geq c$ is equivalent. $(N, 0, 0)$ is an equilibrium because

$$\begin{aligned}\Delta\pi_{12} &= (c - a)(N - 1) \leq 0\\ \Delta\pi_{13} &= (b - a)(N - 1) < 0.\end{aligned}$$

Hence, $(0, N, 0)$ and $(0, 0, N)$ are Nash equilibria as well.
For $n_i, n_j > 0$, $i \neq j$ an equilibrium cannot exist because

$$\Delta\pi_{ij} + \Delta\pi_{ji} = 2a - b - c > 0.$$

Either a player changing from strategy i to j or a player changing from j to i can raise his payoff.

Appendix B

Socially efficient states

We show that $n^* = (3, 3, 3)$ is the unique socially efficient distribution in *game I*, in the sense that it maximizes the (equally weighted) sum of profits in the population. Let

$$\begin{aligned}f(n_1, n_2, n_3) &:= n_1\pi_1(n) + n_2\pi_2(n) + n_3\pi_3(n)\\ &= a\left(n_1^2 + n_2^2 + n_3^2\right) + (b + c)(n_1n_2 + n_1n_3 + n_2n_3).\end{aligned}$$

Since the numbers n_k have to satisfy the restriction $n_1 + n_2 + n_3 = N$, we replace n_3 by $(N - n_1 - n_2)$ and define the function

$$g(n_1, n_2) := f(n_1, n_2, N - n_1 - n_2).$$

Let us for the moment neglect the fact that n_k should be a natural number. Then, we can find socially desirable states by simply maximizing $g(n_1, n_2)$ subject to $n_1, n_2 \geqq 0$, $n_1 + n_2 \leqq N$.

Therefore, we have to solve the first order conditions $\frac{\partial g}{\partial n_k} = 0$, $(k = 1, 2)$, which are explicitly given by

$$\begin{aligned}g_1(n_1, n_2) &= a(4n_1 - 2N + 2n_2) + (b + c)(-n_2 - 2n_1 + N) = 0,\\ g_2(n_1, n_2) &= a(4n_2 - 2N + 2n_1) + (b + c)(-n_1 - 2n_2 + N) = 0.\end{aligned}$$

The second order conditions for a maximum are satisfied. This equation system has an interior solution, $n_1^+ = n_2^+ = \frac{N}{3}$. Since $g(\cdot)$ can be shown to be strictly concave it is guaranteed that the local maximizer (n_1^+, n_2^+) is also a global one.

By the same methods as above we can show that $(3, 3, 3)$ is also the unique socially efficient state in *game II*.

Acknowledgments

We are thankful to Madjid Amir for stimulating discussions and to Andrew Schotter for helpful comments. Financial support of the Sonderforschungsbereich 504 at the University of Mannheim is gratefully acknowledged.

Notes

1. We owe this observation to Andrew Schotter.
2. In the experiments we presented the strategies σ_1, σ_2, and σ_3 as X, Y, and Z and the respective frequencies (n_1, n_2, n_3) as (x, y, z).
3. Note that our data set is rather small.
4. We find a tentative explanation for this difference. If the players alter their strategy randomly in the efficient configuration or one of the almost efficient configurations, the payoffs increase by 29% in game I and by 43% in game II. Thus, the players have more possibilities for increasing their payoff by changing their strategies in game II than they have in game I.
5. The individual payoff change is equal to -2 if a player changes from X to Z, from Y to X, or from Z to Y, and the payoff change is equal to -4 if a player changes from X to Y, from Y to Z, or from Z to X.

References

Berninghaus, S.K. (1984). "A General Existence Theorem for Equilibrium Price Dispersions." *Information Economics and Policy*. 1, 239–266.

Berninghaus, S.K. and Ehrhart, K.-M. (1998). "Time Horizon and Equilibrium Selection in Tacit Coordination Games: Experimental Results." *Journal of Economic Behavior and Organization*. 37, 231–248.

Carlson, J. and McAfee, P. (1983). "Discrete Equilibrium Price Dispersions." *Journal of Political Economy*. 91, 480–493.

Ehrhart, K.-M. (1997). *Kollektivgut-Spiele in diskreter und stetiger Zeit*, Ph.D. Thesis, University of Karlsruhe.

Fudenberg, D. and Kreps, D. (1988). A Theory of Learning, Experimentation, and Equilibrium in Games, mimeo.

Huberman, B.A. and Glance, N.S. (1993). "Evolutionary Games and Computer Simulations." *Proceedings of the National Academy of Science USA*. 90, 7716–7718.

Kalai, E. and Lehrer, E. (1993). "Rational Learning Leads to Nash Equilibrium." *Econometrica*. 61, 1019–1045.

Ledyard, J.O. (1995). "Public Goods: A Survey of Experimental Research." In J.H. Kagel and A.E. Roth (eds.), *The Handbook of Experimental Economics*.

Milgrom, P. and Roberts, J. (1991). "Adaptive and Sophisticated Learning in Repeated Normal Form Games." *Games and Economic Behavior*. 3, 82–100.

Millner, E.L., Pratt, M.D., and Reilley, R.J. (1990). "Contestability in Real-time Experimental Flow Markets." *RAND Journal of Economics*. 23, 584–599.

Rosenthal, R.W. (1973). "A Class of Games Possessing Pure-Strategy Nash Equilibria." *International Journal of Game Theory*. 2, 65–67.

Selten, R. and Berg, C.C. (1970). "Drei experimentelle Oligopolspielserien mit kontinuierlichem Zeitablauf." In H. Sauermann (ed.), *Beiträge zur experimentellen Wirtschaftsforschung*. Tübingen.

Smith, V.L. (1994). "Economics in the Laboratory." *Journal of Economic Perspectives*. 8, 113–131.

van Huyck, J.B., Battalio, R.C., and Beil, R.O. (1990). "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure." *American Economic Review*. 80, 234–249.